Maths for Computing
Tutorial 5

1. Use mathematical induction to prove that $7^{n+2} + 8^{2n+1}$ is divisible by 57 for every nonnegative integer n.

2. Use mathematical induction to prove that for all positive integers n, the number of all subsets of a set of n elements is 2^n.

3. Consider a game in which two players take turns removing any positive number of matches they want from one of two piles of matches. The player who removes the last match wins the game. Show that if the two piles contain the unequal number of matches initially, the first player can always guarantee a win.

4. Prove that for all positive integers n, it is possible to organise a round-robin tournament of n football teams in
 a) $n - 1$ rounds if n is even,
 b) n rounds if n is odd.
A round is a set of games in which each team plays one opponent if n is even, and there is only one idle team if n is odd. A round-robin tournament is a tournament in which any pair of teams meet exactly once. (Use strong induction)

5. Show that it is possible to arrange the numbers $1,2,\ldots,n$ in a row so that the average of any two of these numbers never appears between them. [Hint: Show that it suffices to prove this fact when n is a power of 2. Then use mathematical induction to prove the result when n is a power of 2.]